Prognostication of Hospital Beds Occupancy Using the Kinetic Modelling

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Abstract. It has been observed that the number of patients in a geriatric hospital may be described by mixed-exponential functions. The aim of this paper is to model the kinetics which mimics the flow of patients by a first order differential equations system with constant parameters. This model allows the hospital staff to prognosticate the hospital bed occupancy by simulating different virtual scenarios. Thus, the model may be used to predict short-term future patient movement and therefore aid an optimum decision making concerning the beds requirements. An illustrative application to the case of chronic diseases is also presented.
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1 INTRODUCTION

It has been observed that, after the admission in hospital, the flow of patients appears to be able to be described by first-order differential equations in a similar way the kinetic models do. Patients are initially admitted into acute care consisting of diagnosis, assessment and first medical assistance. The majority of patients are either released and therefore re-enter the community or die following such a period of acute care. A certain number, however, may be considered to be unable to look after themselves and therefore pass from acute into long-stay care where they may remain for a considerable amount of time until they eventually die. Similarly, this basic two-compartment model can be extend to a three-compartment model by integrating in the system an intermediate rehabilitation care between acute and long-stay care, in order to diminish the pressure on the long-stay care. For these models some simplifying assumptions were made, that is all the patients in each compartment are the same, so that the transfer rates between compartments are assumed to be constant. The specific rate constants can be found by fitting the equation giving the cumulative occupancy distribution to an observed cumulative occupancy distribution.

2 THREE-COMPARTMENTS MODEL

The scheme of such a system described above is depicted in the figure below.

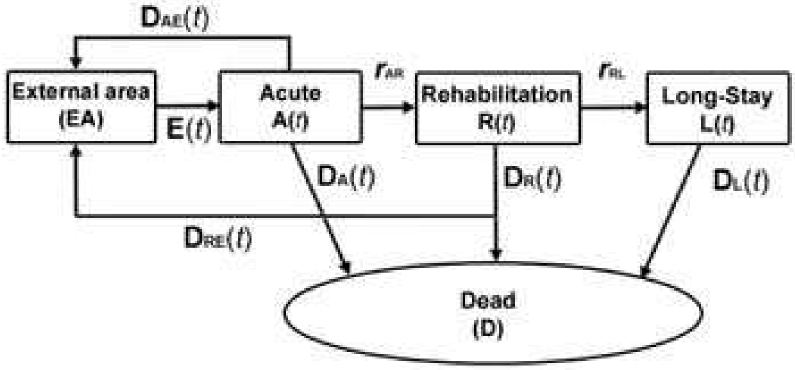

Fig. 1. Three-compartment kinetic model

The patients flow through the hospital is given by the following compartmental features:

- A(t) = number of patients in the Acute compartment at time t;
- R(t) = number of patients in the Rehabilitation compartment at time t;
- L(t) = number of patients in the Long-stay compartment at time t;
- E(t) = number of arrivals at time t;
- $D_{AE}(t)$ = number of patients discharged from Acute at time t;
- r_{AR} = transition rate from Acute to Long-stay;
- $D_A(t)$ = number of deaths in Acute at time t;
- $D_{RE}(t)$ = number of patients discharged from Rehabilitation at time t;
- r_{RL} = transition rate from Rehabilitation to Long-stay;
- $D_R(t)$ = number of deaths in Rehabilitation at time t;
- $D_L(t)$ = number of deaths in Long-stay at time t.

We want to find out how the number of patients changes with time in each compartment by using simple first order differential. These differential equations result from the kinetic model which is our conceptualization of what is happening to the flow of patients in the health care system.

3 COMPARTMENTAL KINETIC NETWORK

A compartmental kinetic network is a network of compartments, such as each node of the network represents a compartment which contains a variable quantity $x_i(t)$ of objects involved in the system. The vector

$$x(t) = (x_1(t), x_2(t), ..., x_n(t))$$

represents the state vector of the system. The system is mass conservative in the sense that the mass balance is preserved inside the system. The transfer rate f_{ij} from a compartment i to another compartment j is considered as a function of the state variables $x(t)$, that is $f_{ij} = f_{ij}(x(t))$. The interactions with the outside (the network environment) are represented by the *inflows* $b_i(t)$, injected from outside into some compartments, and by the *outflows* $e_i(x(t))$ from some compartments to the outside. Such a compartmental system is illustrated in figure below.

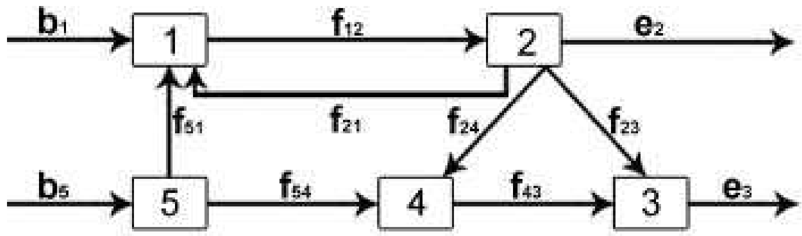

Fig. 2. A standard five-compartments network system

The instantaneous flow balances around the compartments are expressed by the first order system of differential equations:

$$\frac{dx_i}{dt} = \sum_{j \neq i} f_{ji}(x) - \sum_{k \neq i} f_{ik}(x) - e_i(x) + b_i, \quad i = 1, 2, ..., n,$$

known as the system of differential equations of motion underlying the system kinetics.

The flow functions f_{ij} and the outflow functions e_i are defined to be non-negative on the non-negative orthant $[0, \infty)^n$ and, similarly, the inflow functions b_i are defined to be non-negative for all $t \in R_+$.

4 PATIENTS FLOW KINETIC MODEL

In order to develop the series of interdependent differential equations which describes the kinetic of the system, we will split up the three-compartment model into three simple blocks, describing the dynamics of each compartment.

4.1. ACUTE COMPARTMENT DYNAMICS

The corresponding equation of the Acute compartment dynamics is given by:

$$\frac{dA}{dt} = (E - D_{AE} - r_{AR} - D_A) \cdot A.$$

By simply integrating this first order homogeneous differential equation with constant coefficients, we obtain the corresponding solution giving the number of patients in the Acute compartment at time t:

$$A(t) = A_1 \exp\left([E - D_{AE} - r_{AR} - D_A] \cdot t\right), \quad t > 0,$$

where A_1 represents the initial number of patients in the Acute compartment at time $t = 1$.

4.2. REHABILITATION COMPARTMENT DYNAMICS

The corresponding equation is given by:

$$\frac{dR}{dt} = r_{AR} \cdot A - (r_{RL} + D_R) \cdot R.$$

Integrating this inhomogeneous equation using the Lagrange's method of variation of coefficients, we obtain the number of patients in the Rehabilitation compartment at time t:

$$R(t) = \left[R_1 + \int (r_{AR} \cdot A(t))\, dt\right] \cdot \exp\left(-(r_{RL} + D_R) \cdot t\right), \quad t > 0,$$

where R_1 represents the initial number of patients in the Rehabilitation compartment at time $t = 1$ and $A(t)$ is the solution of the first equation.

4.3. LONG-STAY COMPARTMENT DYNAMICS

The corresponding equation is given by:

$$\frac{dL}{dt} = r_{RL} \cdot R - D_L \cdot L.$$

Analogously, we obtain the corresponding solution giving the number of patients in the Long-stay compartment at time t is:

$$L(t) = \left[L_1 + \int (r_{RL} \cdot R(t))\, dt\right] \cdot \exp\left(-D_L \cdot t\right),$$

where L_1 represents the initial number of patients in the Long-stay compartment at time $t = 1$ and $R(t)$ is the solution of the second equation.

Let us remark that these equations may be integrated both by numerical methods and by using the Laplace transform.

5 AN APPLICATION TO A STANDARD GERIATRIC HOSPITAL

A hypothetical example for a typical geriatric hospital providing three compartments: Acute, Rehabilitation and Long-stay care is considered here to illustrate the use of the kinetic model. We have considered a nine-months period of observations in order to predict the number of patients in each compartment for the next short-time period. Fitting the model to data, we obtained the corresponding system of linear differential equations, describing the kinetic of the network. Concretely, we used the regression-based method to estimate the transfer rates between model compartments.

The goodness of fit of the model has been performed using the t-test. Comparing the observed and the expected values in each compartment during a one-year period, we carried out the t-test for independent samples. Thus, the difference between the observed and expected values for every compartment is not statistically significant (p-value ranging between 0.12 and 0.61)

6 DISCUSSION AND CONCLUSIONS

The kinetic model which we have developed has been fitted to a fictitious data set and shown to give an adequate fit for the estimation of the flow of patients through hospital. The hospital staff may consider a number of ways in which the inflows, the outflows and the flows between compartments may change. Using this model to simulate the system under these new circumstances, that is an *in vitro* "What-if ?" type analysis, it may consider the effects of enforced changes to the system without actually applying the changes to the real system. Once the model has been applied to the data, it may then be used as a tool to theoretically consider possible changes to the system links without incurring the time and expense of actually carrying out the changes. Such kinetic models, picturing the dynamics behind the flow of patients, enable an experienced hospital planner to facilitate better decision making concerning bed requirement planning.

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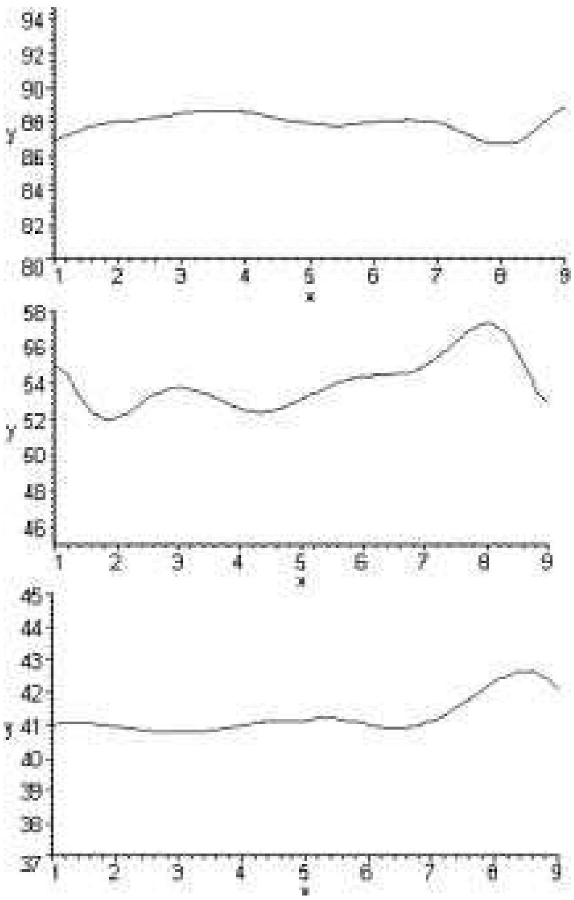

Fig. 3. The dynamics of the flow of patients

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